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CALCULATING THE GREY LINE

By Greg Baker

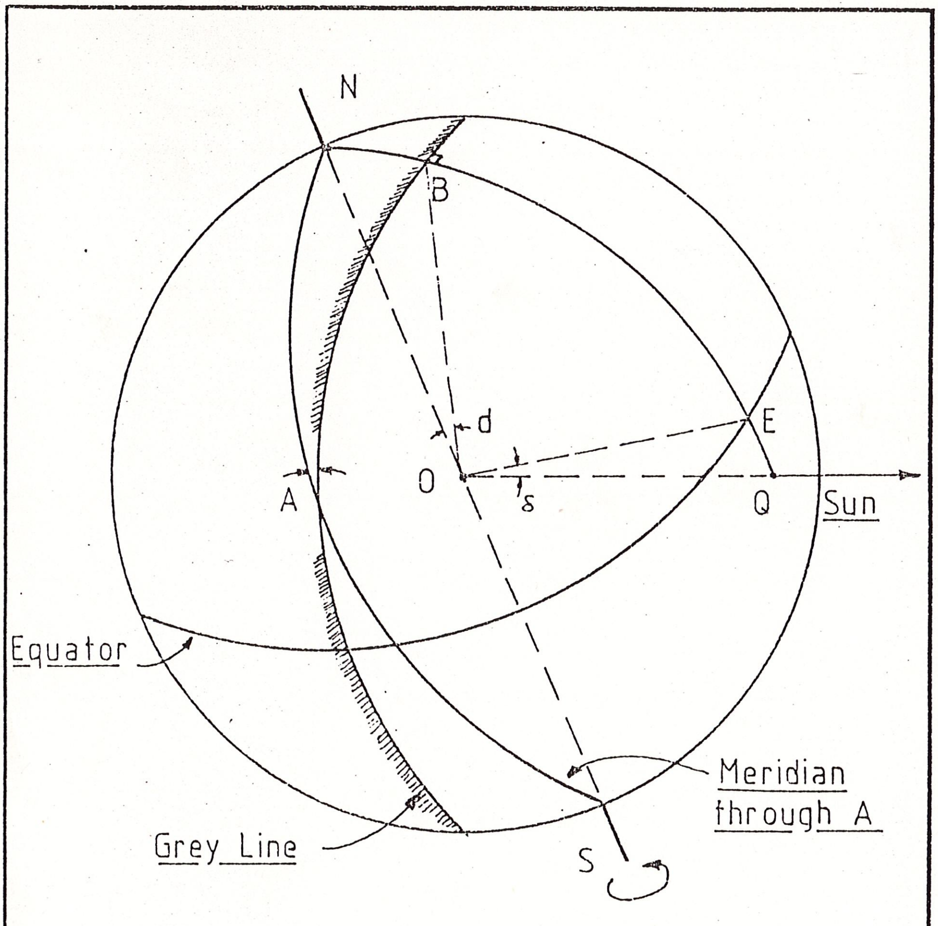

Fig. 1

As the earth spins on its axis, there is always one hemisphere in sunlight and one in shadow. The junction between these two hemispheres - day and night - is a great circle which is called the "grey line". A zone of undefined width along the grey line is called they "grey zone". The grey zone is of interest largely because here there is a fairly abrupt change in the ionosphere. For example, the D-layer disappears almost completely at sunset, bringing with its passing the rapid build-up of MF DX; the opposite is the case at sunrise.

There is also the well known property that efficient communication is possible between stations both lying in the grey zone. Thus it is of interest to amateurs to know where the grey zone is at any time and to exploit its properties where possible.

The easiest method of finding the grey line is to buy a radio globe of the world such as the "Grey Line Radio Globe" reviewed in Amateur Radio Action, Volume 5, Number 11, or to construct one from an ordinary school kids globe as described in Practical Wireless, March 1984.

A more difficult method, but a more accurate one, is to calculate the grey line. It is a relatively straight-forward matter to calculate for any date the bearings of the grey line as it passes any location. For this is a calculator or set of mathematical tables is sufficient. To calculate **when** the grey line passes a location — sunrise and sunset times — a moderately sophisticated calculator is still sufficient. However, in the long sequence of calculations involved, a home computer is not only quicker and easier, it is likely to be more accurate. Because of this the latter part of the article is directed towards home computers, with a reference for further reading for those with calculators only.

BEARINGS OF THE GREY LINE

In Figure 1, NAS is the meridian through location A on the grey line. Q is the subsolar point, ie the place on the earth's surface which lies in a direct line between the centre of the earth (O) and the sun. Any great circle through Q intersects the grey line at right angles. QBN is that part of one of these great circles which passes through the north geographic pole.

What we want to find is angle A and from it the bearings of the grey line, X, and (180 + X) degrees. These will be the bearings at

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LOW BAND DX

sunrise; at sunset the bearings will be (360 − X) and (180 − X) degrees.

For spherical triangle NAB,

sin A/ sin NB = sin B/ sin NA.

NA and NB, although sides of the triangle, are expressed as the angles these sides subtend at the centre of the earth.

Noting that B = sin 90 = 1, that NA is (90 - latitude A) if we set north latitudes positive and south latitudes negative, and setting NB = d, we get

sin A = sin d / sin (90 − lat A)

= sin d / cos (lat A)

Referring again to Figure 1, we can see that d is the same as QOE, which is called the sun's declination.

We know latitude. The only unknown is declination. VK2KII in ARA Volume 6, Number 9, page 33 gives a rough formula which is probably good enough for most purposes. That formula (modified) is d = −23.4 sin (0.9856 D) where D is the number of days after 21st September. The value 0.9856 D is a value in degrees, not radians. The declination, d, clearly ranges in value from −23.4 to 23.4 degrees.

Note that this is the negative of VK2KII's formula to ensure that the usual sign conventions apply, ie northern declinations positive and southern declinations negative. VK2KII has it the other way around.

Since cos (lat A) is positive regardless of the sign of the latitude, A takes on the sign of declination. That is, south declinations such as shown in Figure 1, produce negative values for A and north declinations produce positive values for A. To get the bearing X, the rule is to set X equal to (360 − A) degrees and subtract 360 if this exceeds 360 degrees. This leads to the two sunrise bearings X and (X + 180) degress and the two sunset bearings (360 − X) and (180 − X) degrees. If any of these exceeds 360 degrees, subtract 360 degrees.

For example, on 3rd May, D = 224 and hence d = 15.3 degrees. At latitude −35 degrees, say, A = 18.7 degrees and X = 341.3 degrees. The sunrise bearings of the grey line are 341.3 and 161.3 degrees; the sunset bearings are 18.7 and 198.7 degrees

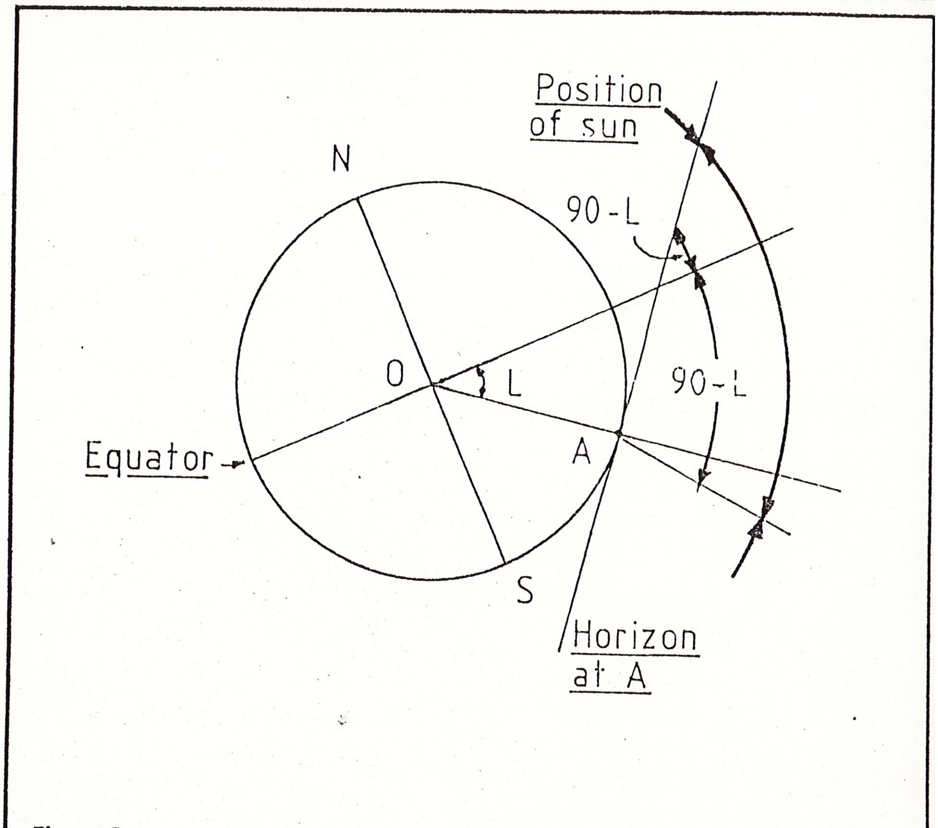

Figure 3

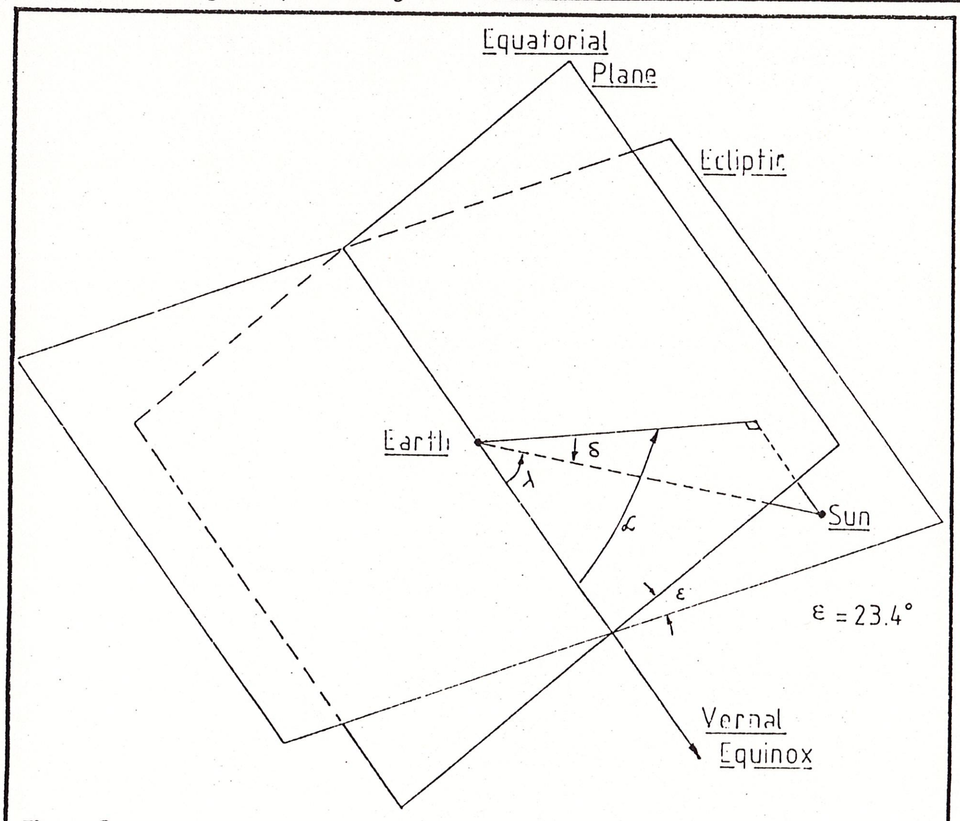

Figure 2

Sunrise and Sunset Times

It is possible to look at a daily newspaper for the times of sunrise and sunset. They don't vary much from day to day, so today's times are probably good enough for tomorrow. The information above is, in these circumstances, sufficient for day to day operation.

However, if you don't buy a daily newspaper, have poor library services, or you want to know, for example, when the grey line will pass the operators in your net across the Tasman, you may want to calculate sunrise and sunset times.

Program GREYLINE described and listed below carries out these calculations accurate to a few minutes. The rest of this section is a description of the procedure followed and can be omitted on first reading. Peter Duffet-

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Smith in his excellent "Practical Astronomy with your Calculator", referenced in full below, has more detail and the interested reader is strongly recommended to get hold of a copy and read the relevant sections.

Sunrise and sunset times depend on where the sun is in relation to the earth and on the location of the point of observation. To find out where the sun is on any date, it is necessary to know how the position of the sun is described by astronomers.

There are two co-ordinate systems used: equatorial co-ordinates and ecliptic co-ordinates.

Equatorial co-ordinates are based on the equatorial plane which is the projection of the plane cutting the earth at the equator. Ecliptic co-ordinates are based on the ecliptic which is the plane in which the earth and sun move. Figure 2 shows both these planes which are at an angle of about 23.4 degrees to one another.

The planes meet in a line which passes through the earth. One direction along this line from earth is used as a reference direction for both co-ordinate systems. It is called the vernal equinox because the sun lies in this direction from earth on 21st March in the northern spring.

In each co-ordinate system the plane and reference direction are used in a manner analagous to the way the plane of the equator and the line from the earth's centre to the Greenwich meridian at the equator are used for our usual geographic co-ordinate system.

Ecliptic longitude (lambda) begins at 0 degrees at the vernal equinox and increases in an anti-clockwise direction in the ecliptic plane to 360 degrees back at the vernal equinox again. The ecliptic latitude (beta) begins at 0 degrees and increases to 90 degrees above and decreases to − 90 degrees below the ecliptic. The ecliptic latitude of the sun is zero always of course.

In equatorial co-ordinates the longitude, called right ascension (alpha) is based on the vernal equinox in a way exactly analagous to that for ecliptic longitude. The angle above or below the equatorial plane is the declination (delta) and is positive above the plane and negative below the plane.

Astronomers have tabulations of various data, including the position of the sun at various times. The position of the sun is given by its ecliptic longitude. Following Duffett-Smith I use the ecliptic longitude at the beginning of 1980 and from this calculate the sun's ecliptic longitude at any time thereafter as:

M + 360/PI.e.sinM + WG

where M = (360/365.2422). d + EG-WG

and D = the number of days since the beginning of 1980

EG = 278.83354 degrees. The ecliptic longitude at the start of 1980.

WG = 282.596403. The ecliptic longitude at perigee, the point where the sun an[d] earth are closest.

e = 0.016718. The eccentricity of the sun[–] earth orbit.

Because the sun is moving relativel[y] rapidly in relation to the earth, the progra[m] calculates the sun's position at the two mid[-]nights straddling the day of interest. It late[r] uses these to get weighted average an[d] hence more accurate sunrise and sunse[t] times.

From the ecliptic co-ordinates, convert t[o] equatorial co-ordinates thus:

Right ascension = tan⁻¹ (sin lambda.cos EP/cos lambda)

Declination = sin⁻¹ (sin EP.sin lambda)

where EP is the angle between the eclipti[c] and the equatorial plane (23.441884 de[-]grees).

Declination gives (i) the bearings of th[e] grey line — as shown above, (ii) whether th[e] sun rises and sets, and (iii) for how long th[e] sun remains above the horizon if it rises an[d] sets. These last two can be seen by refer[-]ence to Figure 3. Consider an observer (A) a[t] south latitude L degrees. Here L is treated a[s] the unsigned latitude, ie the absolute valu[e] of latitude. If the declination of the sun i[s] more than (90-L) degrees north, the observe[r] at A will never see it. If it has a declination o[f] more than (90-L) degrees south, it wil[l] always be above the horizon. If the sun'[s] declination lies in the range (90-L) degree[s] north to (90-L) degrees south, the sun rise[s] and sets.

The length of time it is above the horizo[n] will depend on the latitude of the observe[r] and the declination of the sun relative to (90-L) north and (90-L) south. Algebraically thi[s] time is 2H hours where:

H = (cos ⁻¹ (-tan Latitude.tan delta))/15.

The other equatorial co-ordinate, the righ[t] ascension, leads to the precise period withi[n] the day that the sun is above the horizon. There are several steps. Right ascensio[n] gives local sidereal time (see below) of sun[-]rise and sunset thus:

Rising time = 24 + alpha − H

Setting time = alpha + H.

To understand sidereal times, refer t[o] Figure 4. On 21st March, the sun is at th[e] vernal equinox to an observer on the meri[-]dian through V and it is noon. 23 hours 5[6] minutes later the vernal equinox is agai[n] over the meridian at V. One sidereal ("of th[e] stars") day has passed. Four minutes late[r] again, the sun is over the local meridian at V and one solar day has passed. It is noo[n] again. Because the sidereal day is 23 hour[s] 56 minutes long, sidereal noon falls fou[r] minutes earlier each day than the day before. There are thus approximately 366 siderea[l] days in the 365 solar day year and this i[s] because the earth rotates 366 times in th[e] course of one year not 365. A little ex[-]perimentation with a couple of oranges o[r] tennis balls will show this is the case.

The sidereal rising and setting times nee[d] to be converted into UTC thus:

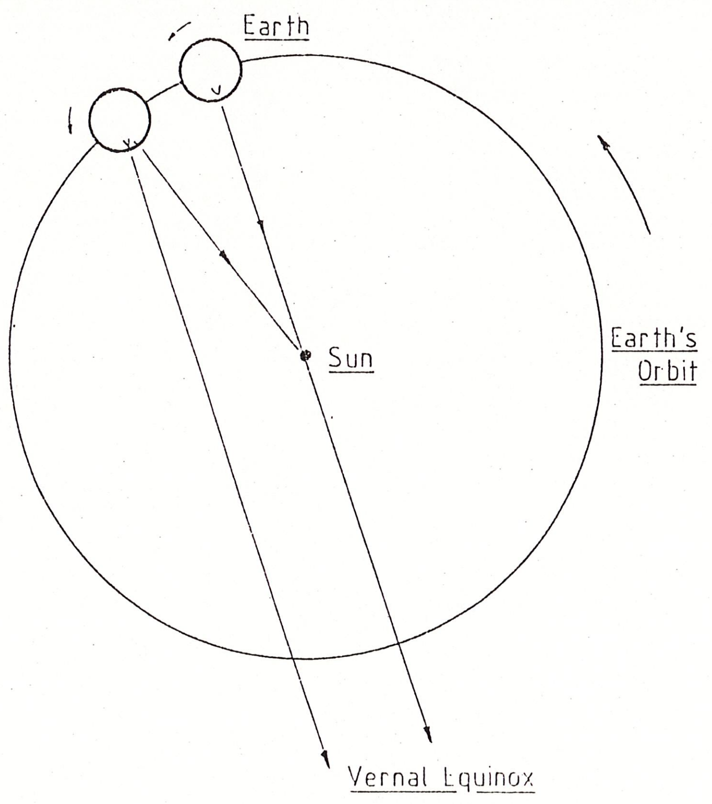

Fig. 4

should be referred to the author. Please include a stamped self addressed envelope and as many details as possible of the problem. Suggestions for improvements and notification of errors will be gratefully received.

Arrays

B(I) The number of days from the beginning of 1980 to the beginning of (1979 + 1)

C(I,J) Right ascension (J = 1) and declination (J = 2) of the sun at the two midnights (I = 1.2) straddling the day of interest.

E(I,J) Number of days in each month (I = 1 to 12) for ordinary (J = 1) and leap years (J = 2)

F(1,J) Bearings of rising (J = 1) and setting (J = 2) sun based on two midnights (I = 1,2) straddling day of interest.

L(I) Ecliptic longitude of the sun at the two midnights (I = 1,2).

Q(I,J) Latitude (I = 1) and longitude (I = 2) of QTH in degrees (J = 1) and minutes (J = 2).
Later in program latitude in decimal form in Q(1,1), longitude in Q(2,1).

S(I,J) Local sidereal times of sunrise (J = 1) and sunset (J = 2) based on the two midnights (I = 1,2).

Greenwich sidereal, and UTC times.

Test Data

The following locations, dates, times and bearings may be useful as test data.

UTC = (t - longitude/15 - d.A + B). 0.99727

The expression in the brackets must be made to lie in the range 0 to 24 hours, by addition or subtraction of multiples of 24, before the multiplication takes place. In the equation, t is local sidereal time, d is the number of days since the beginning of 1980, A = 0.0657098, and B is a constant which is different for each year. The program uses B = 17.37 which is near enough for 1985 and 1986. At around 2400 UTC, this formula does not convert accurately. However, sunrise and sunset times in Oceania should not be affected.

The Program

The program asks for the location latitude and longitude and the date in which you are interested. Latitude and longitude need to be signed. North latitudes are positive; south latitudes are negative. West longitudes are negative; east longitudes are positive. Only sign the degrees, not the minutes. Illegal latitudes and longitudes are signalled and the user asked to re-input. The date is input as DD,MM,YY, eg 22nd April 1985 is 22,04,85 or 22,4,85. Dates must be in the range 1,1,-80 to 31,12,99.

Output form is shown in Figure 5. Sunrise and sunset times are accurate to within a few minutes.

The program runs in the un-enlarged VZ200. Only minor translation should be necessary for other machines. Problems

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	Canberra	Adelaide
Latitude	−35°17′	−34°56′
Longitude	149°13′	138°36′
Date	22.4.84	24.3.84
Sunrise (UTC)	2031	2050
Bearings	345.4°	358.6°
	165.4°	178.6°
Sunset (UTC)	0728	0849
Bearings	194.6°	181.4°
	14.6°	1.4°

References

The basic reference is "Practical Astronomy with Your Calculator", by Peter Duffett-Smith, 2nd Edition, Cambridge University Press, 1982; available in paperback.

Begin with Section 45, Sunrise and Sunse
The analogue methods are in ARA Volum
5, Number 11, and Practical Wireless Marc
1984, and some simple sunrise and sunse
calculations are in Ian VK2KII's article i
ARA Volume 6, Number 9. The Shortwav
Propagation Handbook also addresses th
issue of propagation along the grey lin
(Section 6.8).

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10  DIM B(20),C(2,2),E(12,2),F(2,2),L(2),
    Q(2,2),S(2,2),T(2)
20  DIM S%(2),T%(2)
30  FOR I=1 TO 20
40  READ B(I)
50  NEXT
60  DATA 0,366,731,1096,1461,1827,2192,2557,
    2922,3228
70  DATA 3653,4018,4383,4749,5114,5479,5845,
    6210,6375,6940
80  FOR I=1 TO 12
90  READ E(I,1)
100 E(I,2)=E(I,1)
110 NEXT
120 DATA 31,28,31,30,31,30,31,31,30,31,30,31
130 E(2,2)=29
140 EG=278.83354
150 WG=282.596403
160 PI=3.1415927
170 EC=0.016718
180 DR=57.29578
190 EP=23.441884
230 FL=0
231 PRINT "LATITUDE?   (SIGNED) DEGS, MINS"
240 INPUT Q(1,1),Q(1,2)
250 PRINT "LONGITUDE?  (SIGNED) DEGS, MINS"
260 INPUT Q(2,1),Q(2,2)
270 FOR I=1 TO 2
280 Z=90+(I-1)*90
290 IF ABS(Q(I,1))<=Z THEN 330
300 PRINT "ERROR IN LAT/LONG",Q(I,1),"DEG
    REES",Q(I,2),"MINUTES"
310 PRINT "TRY AGAIN"
320 GOTO 230
330 IF Q(I,2)<0 OR Q(I,2)>=60 THEN 300
340 Q(I,1)=Q(I,1)+SGN(Q(I,1))*Q(I,2)/60
350 NEXT
360 PRINT "DATE?         DD,MM,YY"
370 INPUT DD,MM,YY
380 IF YY<80 OR YY>99 THEN 440
390 IF MM<1 OR MM>12 THEN 440
400 LY=1
410 Y=YY-INT(YY/4)*4
420 IF Y=0 THEN LY=2
430 IF DD>=1 AND DD<=E(MM,LY) THEN 455
440 PRINT "ILLEGAL DATE: TRY AGAIN"
450 GOTO 360
455 D=B(YY-79)+DD
460 FOR I=1 TO MM-1
470 D=D+E(I,LY)
480 NEXT
490 M=360/365.2422*D+EG-WG
500 V=M+360/PI*EC*SIN(M/DR)
510 L(1)=V+WG
520 IF L(1)>=0 THEN 550
530 L(1)=L(1)+360
540 GOTO 520
550 IF L(1)<360 THEN 570
555 L(1)=L(1)-360
560 GOTO 550
570 L(2)=L(1)+0.985647
590 IF L(2)>=360 THEN L(2)=L(2)-360
610 FOR I=1 TO 2
620 Y=SIN(L(I)/DR)*COS(EP/DR)
630 X=COS(L(I)/DR)
640 IF X<>0 THEN 680
650 IF Y>0 THEN C(I,1)=90
660 IF Y<0 THEN C(I,1)=270
670 GOTO 770
680 IF Y<>0 THEN 720
690 IF X>0 THEN C(I,1)=0
700 IF X<0 THEN C(I,1)=180
710 GOTO 770
720 C(I,1)=ATN(Y/X)*DR
730 IF Y>0 THEN 750
740 C(I,1)=C(I,1)+180
750 IF X*Y>0 THEN 770
760 C(I,1)=C(I,1)+180
770 C(I,1)=C(I,1)/15
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```
775  ZZ=SIN(EP/DR)*SIN(L(I)/DR)
776  GOSUB 1390
777  C(I,2)=AS*DR
790  X=SIN(C(I,2)/DR)/COS(Q(1,1)/DR)
800  IF X>-1 AND X<1 THEN 822
810  FL=1
812  GOTO 1262
822  ZZ=X
824  GOSUB 1370
830  F(I,1)=AC*DR
840  F(I,2)=360-F(I,1)
850  X=-TAN(Q(1,1)/DR)*TAN(C(I,2)/DR)
860  IF X<-1 OR X>1 THEN 810
862  ZZ=X
864  GOSUB 1370
870  H=AC*DR/15
880  T(1)=24+C(I,1)-H
890  T(2)=C(I,1)+H
900  FOR J=1 TO 2
910  IF T(J)>24 THEN T(J)=T(J)-24
920  S(I,J)=T(J)
930  NEXT J
935  NEXT I
940  FOR J=1 TO 2
950  T(J)=24.07*S(1,J)/(24.07+S(1,J)-S(2,J))
960  NEXT
970  DE=(C(1,2)+C(2,2))/2
972  ZZ=SIN(Q(1,1)/DR)/COS(DE/DR)
974  GOSUB 1370
980  PS=AC*DR
990  X=0.835608
1000 ZZ=TAN(X/DR)/TAN(PS/DR)
1002 GOSUB 1390
1004 DA=AS*DR
1010 ZZ=SIN(X/DR)/SIN(PS/DR)
1012 GOSUB 1390
1014 Y=AS*DR
1020 DT=240*Y/COS(DE/DR)/3600
1030 FOR J=1 TO 2
1040 T(J)=T(J)+(-1)↑J*DT
1050 FOR I=1 TO 2
1060 F(I,J)=F(I,J)+(-1)↑J*DA
1070 NEXT
1080 T(J)=T(J)-Q(2,1)/15
```

```
1150 DX=D*0.0657098-17.37
1170 T(J)=T(J)-DX
1181 IF T(J)>=0 THEN 1184
1182 T(J)=T(J)+24
1183 GOTO 1181
1184 IF T(J)<24 THEN 1190
1185 T(J)=T(J)-24
1186 GOTO 1184
1190 T(J)=T(J)*0.99727
1192 S%(J)=INT(T(J))
1193 T%(J)=(T(J)-S%(J))*60+0.5
1200 NEXT J
1210 FOR J=1 TO 2
1220 F(1,J)=(F(1,J)+F(2,J))/2-90
1230 F(2,J)=F(1,J)+180
1240 IF F(1,J)<0 THEN F(1,J)=360+F(1,J)
1250 IF F(2,J)>360 THEN F(2,J)=F(2,J)-360
1260 NEXT
1262 CLS
1265 PRINT@0"*****************","GREG BAKE
     MONGARLOWE, 2622"
1266 PRINT "GREYLINE CALCULATOR RESULTS:",
     "*****************"
1270 PRINT "LATITUDE",Q(1,1),"LONGITUDE",
     Q(2,1)
1271 PRINT "DATE:",DD;".";MM;".";YY
1272 IF FL=1 THEN 1290
1273 PRINT "SUNRISE",S%(1);":";T%(1);"UTC"
     "BEARINGS:        ";
1274 PRINT USING "###.#";F(1,1);
1275 PRINT USING "######.#";F(2,1)
1276 PRINT "SUNSET",S%(2);":";T%(2);"UTC",
     "BEARINGS:        ";
1277 PRINT USING "###.#";F(1,2);
1278 PRINT USING "######.#";F(2,2)
1280 PRINT,,,"ANOTHER QTH OR DATE?","TYPE
     'Y' TO CONTINUE"
1282 INPUT Y$
1284 IF Y$<>"Y" THEN 1360 ELSE 230
1290 PRINT "SUN DOES NOT RISE OR SET"
1291 PRINT "HENCE THERE IS NO GREYLINE"
1292 GOTO 1280
1360 END
1370 AC=-ATN(ZZ/SQR(1-ZZ*ZZ))+PI/2
1380 RETURN
1390 AS=ATN(ZZ/SQR(1-ZZ*ZZ))
1400 RETURN
```